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Mapping between Discrete Cosine Transform of Type-VI/VII and Discrete Fourier Transform

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ABSTRACT
In this paper, the mapping between discrete cosine transform of types VI and VII (DCT-VI/VII) of even length N and $(2N – 1)$-point one dimensional discrete Fourier transform (1D-DFT) is presented. The technique used in this paper is the mapping between the real-valued data sequence to an intermediate sequence used as an input to DFT.

Keywords – Discrete Fourier transform, discrete cosine transform, discrete sine transform, mapping.

I. INTRODUCTION

Discrete transforms play a significant role in digital signal processing. Discrete cosine transform (DCT) and discrete sine transform (DST) are used as key functions in many signal and image processing applications. There are eight types of DCT and DST. Of these, the DCT-II, DST-II, DCT-IV, and DST-IV have gained popularity. The DCT and DST transform of types I, II, III and IV, form a group of so-called "even" sinusoidal transforms. Much less known is group of so-called "odd" sinusoidal transforms: DCT and DST of types V, VI, VII and VIII.

The original definition of the DCT introduced by Ahmed *et al*. in 1974 [1] was one-dimensional (1-D) and suitable for 1-D digital signal processing. The DCT has found wide applications in speech and image processing as well as telecommunication signal processing for the purpose of data compression, feature extraction, image reconstruction, and filtering. Thus, many algorithms and VLSI architectures for the fast computation of DCT have been proposed [2]-[7]. Among those algorithms [6] and [7] are believed to be most efficient two-dimensional DCT algorithms in the sense of minimizing any measure of computational complexity. DCT of types I, II, III and IV are extensively used in coding image, video, speech and audio. The so-called "odd-type" DCTs of types V, VI, VII and VIII, however, have not been widely used for purposes of coding image, video, speech and audio data.

Among all the discrete transforms, the discrete Fourier transform (DFT) is the most popular transform, and it is mainly due to its usefulness in very large number of applications in different areas of science and technology. The DFT plays a key role in various digital signal processing and image processing applications [8,9]. Not only it is frequently encountered in many different applications, but also it is computation-intensive.

Recently DST of types VI and VII have surfaced as useful tool in image and video coding. Fast algorithms for computing DST-VII of lengths 4 and 8 have been proposed [10] and these algorithms are currently under consideration for inclusion in ISO/IEC/ITU-T High Efficiency Video Coding (HEVC) standard.

In this paper, the mapping between DCT-VI/VII of even length N and $(2N – 1)$ -point DFT has been established. The procedure used in this paper is the mapping between the real-valued data sequence to an intermediate sequence used as an input to DFT. This mapping technique may find application in image and video coding.

The rest of the paper is organized as follows. The proposed mapping between DCT-VII and DFT is presented in Section-II. Section-III establishes the mapping between DCT-VI and DFT. Conclusion is given in Section-IV

II. MAPPING BETWEEN DCT-VII AND DFT

The 1-D DFT of input sequence $\{ x_n \; ; \; n = 0,1,2,....,N\text{-}1\}$ is defined by

$$Y_k = \sum_{n=0}^{N-1} x_n e^{-\frac{j2\pi kn}{N}}$$

$$for\ k = 0,1,2,\ldots,N-1 \qquad (1)$$

Where $j = \sqrt{-1}$

The DCT of type-VII is defined as

$$X_k^{VII} = \sum_{n=0}^{N-1} x_n COS\left[\frac{(2k+1)n\pi}{2N-1}\right] \qquad for\ k = 0,1,2,\ldots,N-1$$

(2)

The scale factors have been omitted in (1) and (2).
From (1), the real component of DST is written as

$$R[Y_k] = \sum_{n=0}^{N-1} x_n COS\left[\frac{2\pi k n}{N}\right] \quad (3)$$

Replacing k by $2k+1$ and N by $(2N-1)$, (3) can be written as

$$R[Y_{2k+1}] = \sum_{n=0}^{2(N-1)} x_n COS\left[\frac{2\pi(2k+1)n}{2N-1}\right] \quad (4)$$

For N even or odd, (4) can be expressed as

$$R[Y_{2k+1}] = x_o + \sum_{n=1}^{N-1}[x_n + x_{2N-1-n}] COS\left[\frac{2\pi(2k+1)n}{2N-1}\right] \quad (5)$$

For mapping between DCT-VII and DFT, define

$$\begin{cases} x_o = 0 \\ x_n = x_{2N-1-n} \quad for\ n = 1,2,\ldots,N-1 \end{cases} \quad (6)$$

Substituting (6) in (5), we obtain

$$\frac{R[Y_{2k+1}]}{2} = \sum_{n=1}^{N-1} x_n COS\left[\frac{2\pi(2k+1)n}{2N-1}\right] \quad (7)$$

Since from (6) $x_o = 0$, (7) can be written as

$$\frac{R[Y_{2k+1}]}{2} = \sum_{n=0}^{N-1} x_n COS\left[\frac{2\pi(2k+1)n}{2N-1}\right] \quad (8)$$

Taking N as even, (8) can be expressed as

$$\frac{R[Y_{2k+1}]}{2} = \sum_{n=0}^{\frac{N}{2}-1} x_n COS\left[\frac{\pi(2k+1)2n}{2N-1}\right] + \sum_{n=0}^{\frac{N}{2}-1} x_{N/2+n} COS\left[\frac{\pi(2k+1)2\left(\frac{N}{2}+n\right)}{2N-1}\right] \quad (9)$$

Define another sequence u_{2n} and u_{N+2n} given by

$$\begin{cases} x_n = u_{2n} \quad for\ n = 0,1,2,\ldots,\frac{N}{2}-1 \\ x_{N/2+n} = u_{N+2n} \quad for\ n = 0,1,2,\ldots,\frac{N}{2}-1 \end{cases} \quad (10)$$

Substituting (10) in (9), we get

$$\frac{R[Y_{2k+1}]}{2} = \sum_{n=0}^{\frac{N}{2}-1} u_{2n} COS\left[\frac{\pi(2k+1)2n}{2N-1}\right] + \sum_{n=0}^{\frac{N}{2}-1} u_{N+2n} COS\left[\frac{\pi(2k+1)(N+2n)}{2N-1}\right] \quad (11)$$

For even N, the DCT-VII defined in (2) can be expressed as

$$X_k^{VII} = \sum_{n=0}^{\frac{N}{2}-1} x_n COS\left[\frac{\pi(2k+1)n}{2N-1}\right] + \sum_{n=0}^{\frac{N}{2}-1} x_{N/2+n} COS\left[\frac{\pi(2k+1)\left(\frac{N}{2}+n\right)}{2N-1}\right] \quad (12)$$

The above expression shows that the RHS of the real part of 1D-DFT given by (11) is in the form of DCT-VII. Hence,

$$\frac{R[Y_{2k+1}]}{2} = X_k^{VII} \quad (13)$$

The above expression shows the mapping between DCT-VII and DFT.

III. MAPPING BETWEEN DCT-VI AND DFT

The DCT of type-VI for input sequence {xn ; n = 0,1,2,....,N-1} is defined by

$$X_k^{VI} = \sum_{n=0}^{N-1} x_n COS\left[\frac{k(2n+1)\pi}{2N-1}\right] \quad for\ k = 0,1,2,\ldots,N-1 \quad (14)$$

The scale factor has been omitted in (14).

Replacing N by $2N-1$ in (3), we obtain

$$R[Y_k] = \sum_{n=0}^{2(N-1)} x_n COS\left[\frac{2\pi k n}{2N-1}\right] \quad (15)$$

For N even or odd, (15) can be written as

$$R[Y_k] = x_o + \sum_{n=1}^{N-1}[x_n + x_{2N-1-n}] COS\left[\frac{2\pi k n}{2N-1}\right] \quad (16)$$

For mapping between DCT-VI and DFT, define

$$\begin{cases} x_o = 0 \\ x_n = x_{2N-1-n} \quad for\ n = 1,2,\ldots,N-1 \end{cases} \quad (17)$$

Substituting (17) in (16), we get

$$\frac{R[Y_k]}{2} = \sum_{n=0}^{N-1} x_n COS\left[\frac{2\pi k n}{2N-1}\right] \quad (18)$$

For even N, define another sequence

$$x_n = -v_{(N-n-1)} \quad for\ n = 0,1,2,\ldots,N-1 \quad (19)$$

Substituting (19) in RHS of (18), we get

$$\frac{R\left[Y_k\right]}{2} = \sum_{n=0}^{N-1} v_n \cos\left[\frac{k(2n+1)\pi}{2N-1}\right]$$
(20)

Since the RHS of the above expression is in the form of DCT-VI defined by (14), we have

$$\frac{R\left[Y_k\right]}{2} = X_k^{VI}$$
(21)

As the LHS of (21) represents the real part of DFT, the mapping between DCT-VI and DFT is established by this equation.

IV. CONCLUSION

In this paper, the mapping between DCT-VI/VII of even length N and $(2N - 1)$ -point DFT has been established using an intermediate sequence. The performance DST can be compared to that of DCT and therefore, DST may be considered as a viable alternative to DCT. Recently DST of types VI and VII have surfaced as useful tool in image and video coding. Fast algorithms for computing DST-VII of lengths 4 and 8 have been proposed [10] and these algorithms are currently under consideration for inclusion in ISO/IEC/ITU-T High Efficiency Video Coding (HEVC) standard. Like DST-VI/VII, the proposed mapping technique for DCT-VI/VII may find application in image and video coding.

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